# Comparative study of high order statistics estimators

***Abstract:*** **To characterize and process a signal, many high order statistics are used by the signal processing researchers. Specific features of the data (temporal, stationary) and real time applications require the development of new estimators. In this paper, we study some estimators of high order moment and cumulant using adapted to different kind of signals.**

***Key words:*** **Estimators, high order statistic, moment, cumulant, temporal and non-stationary data.**

## 1. INTRODUCTION

## 2. THEORETICAL BACKGROUND

$$\phi_X \; t \; = \int_{-\infty}^{+\infty} \quad itx \;\; p_X \;\; x \;\; dx$$

$$\Phi_X\left( \quad \right) =$$

$$\varphi_X \;\; t \;\; = \quad \phi_X \;\; t$$

$qth$

$\{v \ v \quad v_p \quad \leq p \leq q\}$

$v_i \in \{ \quad q-p+ \ \}$ $\quad \sum_{i=}^{p} v_i = q$

$q$

$$Cum \quad X \ = \mu \ - \ \mu \ \mu \ - \ \mu \ + \ \ \mu \ \mu \ - \ \mu$$

$$Cum \quad X \ = \mu \ - \ \mu$$

## 3. HIGH ORDER STATISTICS ESTIMATORS

### 3.1 Arithmetic estimators

$N$ $\quad x_i$ $\quad X$

$qth$

$$\mu_q = \frac{}{N} \sum_{i=}^{N} x_i^q$$

$X$ $\quad N$

$$\mu_q \ = \frac{}{N} \ \mu_{\ q} - \mu_q$$

$qth$

$$\widehat{Cum}_q \ X \ = \sum_{p=}^{q} - \ ^{p} \ p- \quad \mu_v \ \mu_v \quad \mu_{v_p}$$

$\mu_v \ \mu_v \quad \mu_v \ \mu_v$

$$E\left[\widehat{Cum}_q \ X \ \right] = \sum_{p=}^{q} \frac{- \quad ^{p}}{N^{p-}} \ p- \quad \begin{pmatrix} \mu_q \\ + \ N- \quad \mu_{v_k} \ \mu_{v_k} \\ + \ N- \quad \mu_{v_k} \ \mu_{v_k} \ \mu_{v_k} \\ + \ \ + \ N- \quad ^{p-} \ \mu_v \quad \mu_{v_p} \end{pmatrix}$$

$$\sum_{i=}^{p} v_i = q$$

$\mu_{v_k} \ \mu_{v_k} \quad v_k \quad v_k$

$\{v \ v \quad v_p\}$

$$\widehat{Cum}_q \ X \ = \sum_{p=}^{q} c_p \ - \ ^{p} \ p- \quad \mu_v \ \mu_v \quad \mu_{v_p}$$

$c_p$

$v_i$

$$\widehat{Cum} \quad X \ = a\mu \ - \ b\mu \ \mu \ - \ c\mu \ + \ \ d\mu \ \mu \ - \ e\mu$$

$$\begin{cases} a = \dfrac{N \ \ + N \ - \ \ N+}{N- \quad N- \quad N-} & b = \dfrac{N \quad N \ - \ \ N+}{N- \quad N- \quad N-} \\ c = \dfrac{N \ N \ \ - N-}{N- \quad N- \quad N-} & d = \dfrac{N \quad N-}{N- \quad N- \quad N-} \\ e = \dfrac{N}{N- \quad N- \quad N-} \end{cases}$$

$$\widehat{Cum} \quad X \ = \frac{N+}{N-} \mu \ - \ \frac{N}{N-} \mu$$

$qth$

$$\mu_q \ k \ = \frac{}{k} \sum_{i=}^{k} x_i^q = \frac{k- \quad \mu_q \ k- \quad + x_k^q}{k}$$

$$\mu_q \quad = x^q$$

$q$

$\mu_i \ k- \quad i \in \quad q$

### 3.2 Exponential estimators

$$\mu_q = \quad -\lambda_q \sum_{i=}^{N} \lambda_q^{N-i} x_i^q$$

$$< \lambda_q <$$

$$\mu_q \;\; k \;\; = \lambda_q \mu_q \;\; k - \quad + \quad -\lambda_q \;\; x_k^q$$

$$E\left[\mu_q\right] = \quad -\lambda_q^N \;\; \mu_q$$

$$\lambda_q$$

$$\mu_q = \frac{-\lambda_q}{-\lambda_q^N} \sum_{i=}^{N} \lambda_q^{N-i} x_i^q$$

$$\mu_q \;\; k \;\; = \frac{}{-\lambda_q^k} \left( \lambda_q \quad -\lambda_q^{k-} \;\; \mu_q \;\; k - \quad + \quad -\lambda_q \;\; x_k^q \right)$$

$$\widehat{Cum} \quad X \quad k \;\; = \widehat{Cum} \quad X \quad k - \quad + \quad -\gamma \;\; H_k \;\; \widehat{Cum} \quad X \quad k -$$

$$\gamma$$

$$H_k \;\; x \;\; = x_k - \quad x_k \mu \quad k - \quad -x$$

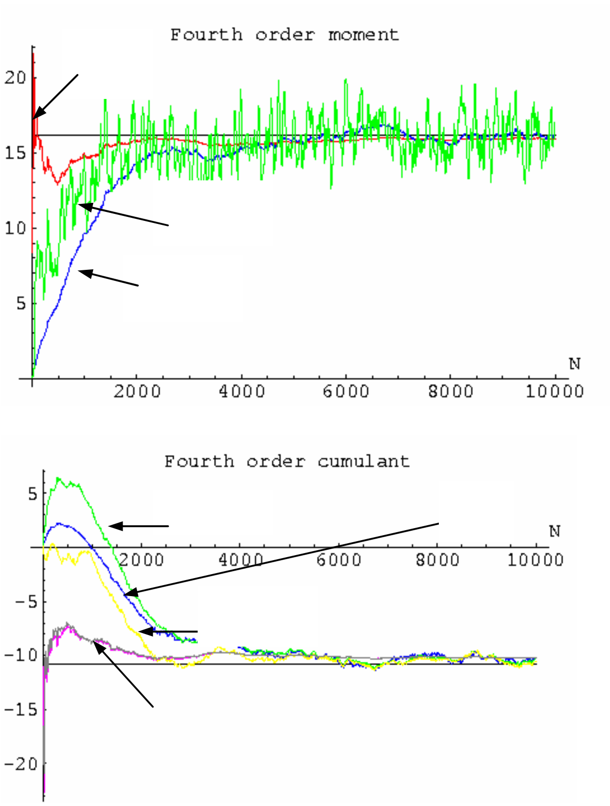


*Figure 1. Fourth order moment and cumulant estimators of a centered and stationary signal (type white noise), the forgotten factor is 0.999 for all moments and* γ = [1].

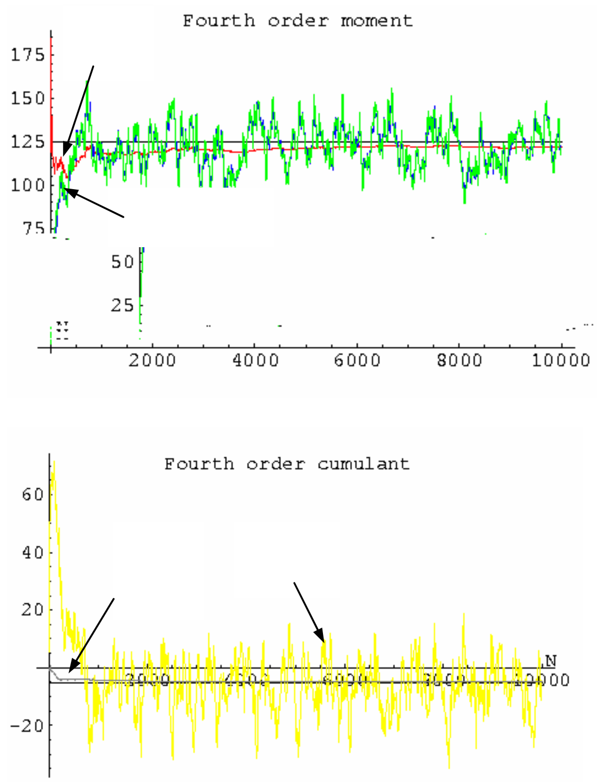


*Figure 2. Fourth order moment and cumulant estimators of a non-centered and stationary signal (type white noise), the forgotten factor was 0.99 for all moments and* γ = [1].