

# Effective ATR Algorithms Using Information Fusion Models

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**Abstract** – Several types of classifiers have been developed in order to extract the information for the automatic target recognition (ATR). We have noted that these performances are different according to the classifier and the radar target. We propose in this article three approaches of information fusion in order to outperform three radar target classifiers. These three techniques of fusion are the Sugeno's fuzzy integral, the possibility theory and the Dempster-Shafer theory. In this application, we show that the best performance is achieved by the Dempster-Shafer theory.

**Keywords:** ! " # " "\$ % & ' ( & ) \* #' + ) " ) ,

## 1 Introduction

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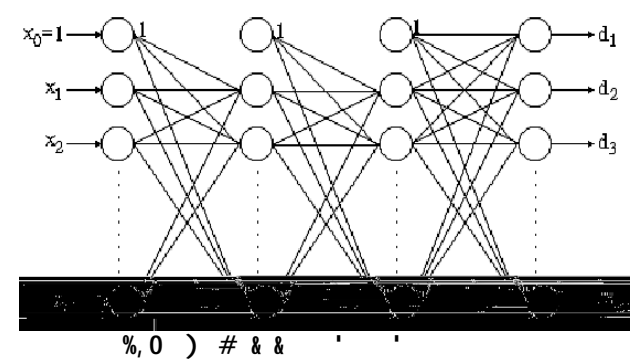
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## 2 Classifiers

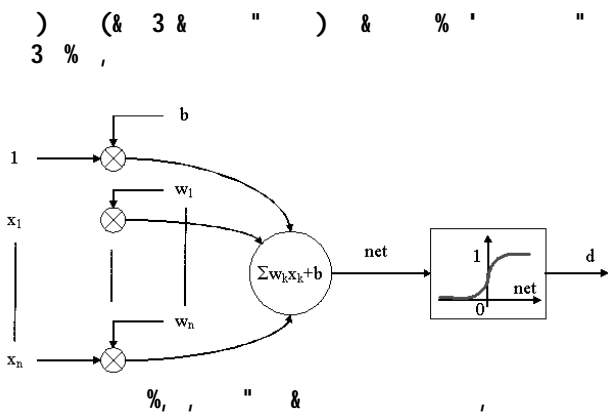
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### 2.1 Multilayer perceptron classifier

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$\varepsilon = -\sum_{j=0}^m -d_j - o_j$

$$w_{ij} - t + 0 = w_{ij} - t + \eta \delta_j - t \cdot o_i - t$$

$$\begin{cases} \delta_j = co_j / 0 - o_j / d_j - 1 \\ \delta_j = co_j / 0 - o_j - 1 \sum_l \delta_l w_{lj} \end{cases}$$

$o_k - x = \#_j \{o_j - x\} \Rightarrow x \in C_k$

## 2.2 Fuzzy K Nearest Neighbor Classifier

$K_j - x = \{x_n | x_n \in C_j, x_n \in V_K - x\}$

$$K_k - x = \#_{j=0}^m [K_j - x] \Rightarrow x \in C_k$$

$o_k - x = \#_j \{o_j - x\} \Rightarrow x \in C_k$

$u_{jt} = \frac{K_j^{-1}}{K_F} \quad K_j^{-1} = \{x_n^{-j} \mid x_n^{-j} \in V_{K_F}(x_t)\} \quad -<$

$$u_{jt} = \frac{K_j^{-1}}{K_F} \quad K_j^{-1} = \{x_n^{-j} \mid x_n^{-j} \in V_{K_F}(x_t)\} \quad -<$$

$o_k - x = \#_j \{o_j - x\} \Rightarrow x \in C_k$

$$o_j - x = \sum_{i=0}^K (u_{jt} \mathbb{B} \|x - x_i\|^{\delta}) \mathbb{B} \sum_{i=0}^K (\mathbb{O} \mathbb{B} \|x - x_i\|^{\delta})$$

$o_k - x = \#_j \{o_j - x\} \Rightarrow x \in C_k$

$$o_k - x = \#_j \{o_j - x\} \Rightarrow x \in C_k$$

$o_k - x = \#_j \{o_j - x\} \Rightarrow x \in C_k$

## 2.3 SART classifier

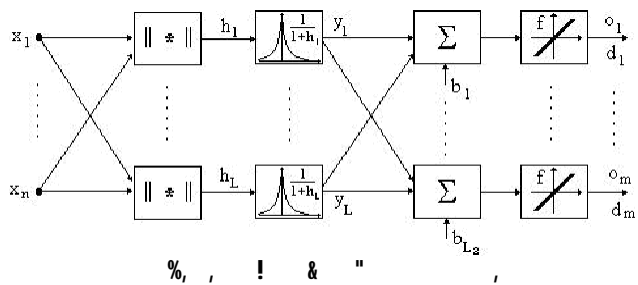
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### 3 Fusion models

#### 3.1 Sugeno's fuzzy integral

$$\chi = \int_Q h \cdot q \cdot \circ g \cdot \cdot = \#_{A \subseteq Q} \left[ \#_{q \in A} (h \cdot q \cdot \circ g \cdot A \cdot) \right]$$

$$= \#_{\alpha \in I} \left[ \#_{\alpha} (g \cdot h \cdot \alpha \cdot) \right]$$

$$4) \quad h_\alpha = \{q \mid h \cdot q \cdot > \alpha\}$$

$$h \cdot q \cdot 2 \quad \dots \quad 5 \quad ( \quad ) \quad \& \quad " \quad q$$

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$$\quad ) \quad \# \quad \# \quad h \cdot q \cdot \quad \# \quad ) \quad ) \quad \%$$

$$4) \quad ) \quad ) \quad ' \quad h \quad " \quad ( \quad \& \quad ) \quad \& \quad \# \quad " \quad )$$

$$( \quad A_i \quad g \cdot A \cdot \quad ' \quad ) \quad \# \quad ' \quad " \quad ) \quad \%$$

$$\& \quad " \quad 2 \quad 3 \quad \& \quad 4 \quad ) \quad \%$$

$$\quad ) \quad ' \quad g_i \quad , \quad ) \quad \%$$

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$$h \cdot q_0 \cdot \geq h \cdot q \cdot \geq \dots \geq h \cdot q_N \cdot H \quad " \quad ) \quad \& \quad \%$$

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$$\chi = \int_Q h \cdot q \cdot \circ g \cdot \cdot = \#_{i=0}^N \left[ \# (h \cdot q_i \cdot \circ g \cdot A_i \cdot) \right]$$

$$4) \quad > \quad A_i = \{q_0 \quad q \quad \dots \quad q_i\}$$

$$g \cdot A_i \cdot \quad ( \quad \& \quad \& \quad 3 \quad 4 \quad \%$$

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$$\begin{cases} g \cdot A_0 \cdot = g(\{q_0\}) = g^0 \\ g \cdot A_i \cdot = g^i + g \cdot A_{i-1} \cdot + \lambda \cdot g^i \cdot g \cdot A_{i-1} \cdot \quad i = \overline{1, \dots, N} \end{cases}$$

#### A. Training stage

$$0. \quad 9 \quad ) \quad ' \quad " \quad \# \quad " \quad ) \quad \& \quad " \quad ($$

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$$g_j^i = g_j(\{q_i\}) \quad ( \quad ) \quad \%$$

$$) \quad \& \quad " \quad q_i \quad " \quad ) \quad \& \quad j_i$$

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$$g \cdot Q \cdot = 0 \Rightarrow \lambda + 0 = \prod_{i=0}^N -0 + \lambda g_j^i \cdot \quad -00.$$

#### B. Classification stage

$$0. \quad \& \quad \& \quad ) \quad ' \quad o_j^i = h_j \cdot q_i \cdot \quad j=0 \quad m \quad i=0 \quad N \quad " \quad ) \quad N$$

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$$\chi_j = \#_{i=0}^N \left[ \# (o_j^i \cdot g_j \cdot A_i \cdot) \right] \quad -0 \cdot$$

$$\cdot \quad * \quad ) \quad \# \quad \# \quad ( \quad ) \quad ' \quad " \quad ) \quad \%$$

$$\cdot \quad " \quad \#_{j=0}^m \chi_j \geq \chi \quad k = \#_{j=0}^m \chi_j \Rightarrow x \in C_k$$

$$( \quad " \quad \#_{j=0}^m \chi_j < \chi \Rightarrow x \in C_k$$

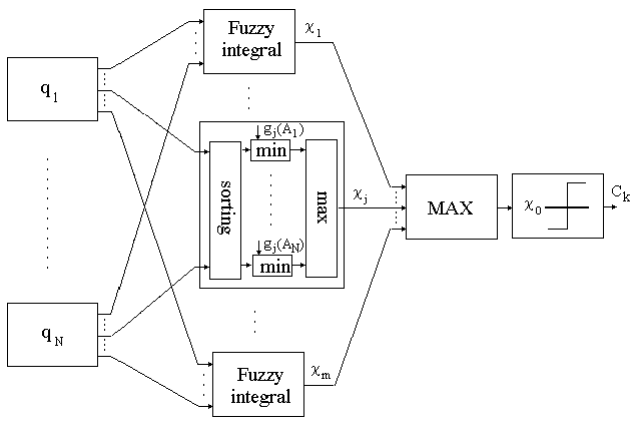
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$\mu, \mu_j, \mu_j(A_j)$

### 3.2 Possibility theory

Let  $D$  be a set,  $\pi: D \rightarrow [0, 1]$  a possibility function. For any  $A \subseteq D$ , the possibility measure is defined as  $\Pi(A) = \max_{x \in A} \pi(x)$ . The possibility distribution is  $\pi(x) = \max_{A \in \mathcal{A}} \Pi(A) \cap x$ .

$$\pi(x) = \max_{A \in \mathcal{A}} \Pi(A) \cap x$$

Let  $\alpha_j$  be a possibility distribution. The possibility measure of a set  $A$  is  $\Pi(A) = \max_{x \in A} \alpha_j(x)$ . The possibility distribution is  $\alpha_j(x) = \max_{A \in \mathcal{A}} \Pi(A) \cap x$ .

$$\forall A \in \mathcal{A} \quad \Pi(A) = \max_{x \in A} \alpha_j(x)$$

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### 3.3 Dempster-Shafer theory

Let  $C_1, \dots, C_N$  be a partition of a set  $D$ . Let  $\alpha_j$  be a possibility distribution. The possibility measure of a set  $A$  is  $\Pi(A) = \max_{x \in A} \alpha_j(x)$ . The possibility distribution is  $\alpha_j(x) = \max_{A \in \mathcal{A}} \Pi(A) \cap x$ .

$$\sum_{A \subseteq D} m(A) = 1$$

Let  $\alpha_j$  be a possibility distribution. The possibility measure of a set  $A$  is  $\Pi(A) = \max_{x \in A} \alpha_j(x)$ . The possibility distribution is  $\alpha_j(x) = \max_{A \in \mathcal{A}} \Pi(A) \cap x$ .

$$\begin{cases} m_j^i(\{C_i\}) - x = \alpha_j R_j p - q_j \mathbb{B}_{C_i} \cdot \mathbb{B}_0 + R_j p - q_j \mathbb{B}_{C_i} \dots \\ m_j^i(\{C_i^c\}) - x = \alpha_j \mathbb{B}_0 + R_j p - q_j \mathbb{B}_{C_i} \dots \\ m_j^i - D - x = 0 - \alpha_j \end{cases}$$

Let  $\alpha_j$  be a possibility distribution. The possibility measure of a set  $A$  is  $\Pi(A) = \max_{x \in A} \alpha_j(x)$ . The possibility distribution is  $\alpha_j(x) = \max_{A \in \mathcal{A}} \Pi(A) \cap x$ .

$$m_{-i} = \bigoplus_{i=0}^{N-1} \bigoplus_{j=0}^{m-1} m_j^{-i} \dots$$

Let  $\alpha_j$  be a possibility distribution. The possibility measure of a set  $A$  is  $\Pi(A) = \max_{x \in A} \alpha_j(x)$ . The possibility distribution is  $\alpha_j(x) = \max_{A \in \mathcal{A}} \Pi(A) \cap x$ .

$$K = \sum_{B_0 \cap B} \prod_{i=0}^N m_i \cdot B_i < 0$$

Let  $\alpha_j$  be a possibility distribution. The possibility measure of a set  $A$  is  $\Pi(A) = \max_{x \in A} \alpha_j(x)$ . The possibility distribution is  $\alpha_j(x) = \max_{A \in \mathcal{A}} \Pi(A) \cap x$ .

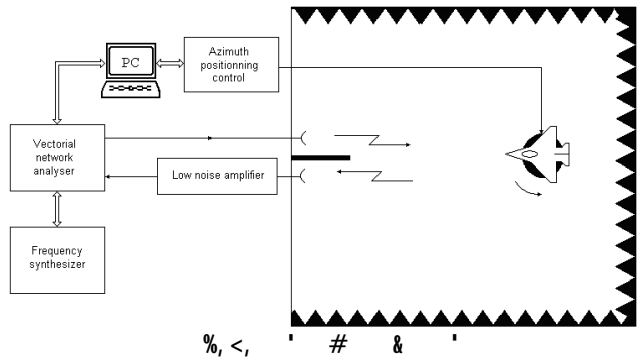
$$m-A = \sum_{B_0 \cap B} \prod_{i=0}^N m_i - B_i \quad - 0.$$

$$m-N = K$$

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#### 4 Database and simulation results

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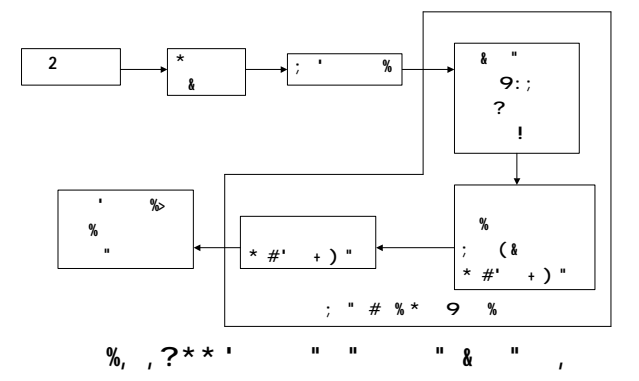


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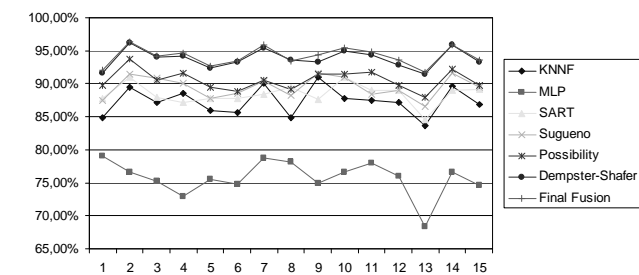
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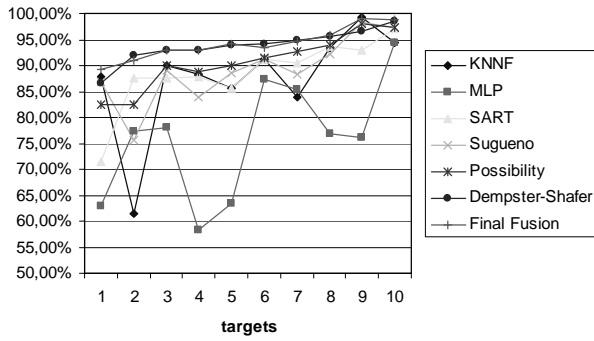
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### 5 Conclusions and future works

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